Lecture 2: *Time, tense, and tense-logic*

Tero Tulenheimo

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1 Change

Time and change are strongly related notions:

(a) Time presupposes change epistemically.

(b) Change presupposes time logically.

Consider [b]: Genuine change resolves into two elementary changes. Think, for illustration, discrete time and write $\alpha T \beta$ for “$\alpha$ and next $\beta$”: $p T q$ means $p T \neg p$ and $\neg q T q$. This is a kind of coordinate representation of change in terms of the base \{p, q\}. Were it not for time, change would entail contradiction: $(p \land \neg p), (\neg q \land q)$.

* Notes for the second lecture of the course “Tense and Logic,” at the Department of Philosophy, University of Helsinki, September 13, 2006. The notes are deliberately sketchy.
Suppose changes occur. We must describe changes by arranging contradictorily related states in an order of succession (or, in the very least, they must be described as being non-simultaneous), else we have contradictions. E.g., suppose there occurs change involving \((p \lor q), (\neg p \land \neg q)\). We must posit one of the two as temporally prior to the other, to avoid a contradiction in the description.

Consider \([a]\): *Empty time and its variants*. Suppose world comes to a complete standstill, no further change takes place. This involves, *inter alia*, that all life becomes extinct. Suppose this occurs tomorrow at 10 o’clock. Do we say that time comes to an end at 10 o’clock tomorrow? That after 10 o’clock tomorrow there is no time? Presumably yes. What the example does in effect, is to deny the applicability of temporal concepts. Figuratively: time no longer exists — however, “no longer” applies only to something which – when existing – exists in time. But the existence or non-existence of time is not itself temporal.

In a world without change the concept of time would have no application. If we think away change from the world, we cannot think of the world as existing in time. On the level of phenomena, what we experience or witness are changes. Change is epistemically prior to time. Time itself is not experienced or witnessed. “Experience of time” is really experience of things (changes, events) *in* time. Von Wright points out that this is a way of understanding Kant’s idea that time is a form of apprehending phenomena (“Time, Change, and Contradiction,” 1969, *Philosophical Papers Vol. II*: 126).

## 2 Temporal order

*Temporal order of time points*. Suppose time points are construed as equivalence classes of simultaneous events. If \(t = [e]_\sim\) and \(t' = [e']_\sim\), then we say \(t < t'\) whenever the event \(e\) precedes the event \(e'\). This definition makes sense since it is not dependent on the choice of the representatives of the equivalence classes \([e]_\sim\) and \([e']_\sim\): by definition any elements in each class are mutually simultaneous. In practice, we will simply consider relations like *later than*, *earlier than* and *simultaneity* directly as relations among time points.

*Possible properties of temporal order*: irreflexive, transitive (these two virtually necessary\(^1\) for the relation *earlier than*); linear, branching, circular

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\(^1\) Though irreflexivity must be denied if circularity is taken seriously.
[speculations of ‘perpetual return’], discrete, dense, continuous.

Note that some temporal expressions assert / presuppose certain properties of the time-medium, e.g. “and next”: Saying ‘p and next q’ at a time point t presupposes / asserts that the time point t has an immediate successor.

3 A-series and B-series

McTaggart (1866-1925) is well known for having pointed out\(^2\) that there are two ways of viewing temporal order: time as forming (i) an A-series, and time as forming (ii) a B-series. McTaggart explains that the A-series is the order of positions in time as past, present and future, whereas the B-series is the order of positions in time as earlier or later.

As P. Geach\(^3\) and C. D. Broad\(^4\) both separately point out, it is also useful to make a distinction between A-characteristics and B-characteristics, not least because the terminology thus yielded is more rigorous than the talk about a series, notably in the case of the A-series. As Geach (1976, p. 90) puts it:

[A]s McTaggart presents this idea [of an A-series] it is not at all clear why he speaks of a series, or what is supposed to be the ordering relation of the series.

\(\text{Being past, being present, being future, being yesterday and having happened ten years ago}\) are A-characteristics: they are all characteristics (qualities or relations) that can only be ascribed to entities from a fixed point of view which is taken to be the present. By contrast, \(\text{being earlier than some event or being later than some event, lasting an hour and being ten years apart in birthdays}\) are B-characteristics: possessing such characteristics is not relative to any fixed now-point. The idea behind the distinctions is obvious: the A-series and A-characteristics are indexical and perspectival by nature, while the B-series and B-characteristics are absolute.

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In the notes for Lecture 1, Subection 1.3, it was noted that certain sentences such as “Rome is smaller than London” or “Socrates is sitting” can be seen from two different vantage points: one option is to think they do not by themselves serve to express a complete proposition or serve to make a complete statement: to yield such a statement they must be supplemented by an explicit time indication. The other option is to think indeed they serve to express a complete proposition, even if the truth-value of the proposition expressed varies with time. The first option would construe such sentences as being of the form $Pt$; the other as being of the form $\lambda t Pt$, that is, as attributes rather than attributions of times.\(^5\) (An analogous distinction would be: $x$ is white vs. whiteness.)

Accordingly, the statements understood according to the first option would, when supplemented with a time indication, yield ‘eternal sentences’ of the form

\[(1) \, p \text{ at } t.\]

By contrast, statements construed on the latter model would be as such full-fledged propositions, of the form

\[(2) \, p \text{ (now)}.

To make reference to these two interpretive options easier, let us label sentences of the type (1) $B$-sentences, and sentences of the type (2) $A$-sentences. Then anyone committed to A-sentences will by definition consider them as capable of expressing complete sentential meanings. He or she must certainly also accept that B-sentences do so. From the vantage point of an adherent of A-sentences, B-sentences then differ from A-sentences in that their truth-value does not change with time. By contrast, anyone committed to B-sentences at the expense of A-sentences will by definition think that A-sentences do not as such express complete sentential meanings. From such a perspective, what an A-sentence states (and not merely its truth-value) varies with time.

\(^5\) If $Px$ is a one-place propositional function, $\lambda x Px$ stands for the property of being $P$. Hence the expression ‘$\lambda x Px$’ can be applied to an individual term, and in particular if $c$ is a constant, then $\lambda x Px(c)$ is true iff $P(c)$ is true.
4 Token-reflexivity

As was seen in the notes of Lecture 1, Subsection 1.2, many descriptions of individuals are constructed by reference to other individual descriptions: *Napoleon’s mother, Mary’s house*. An important subclass of such individual descriptions are those in which the individual referred to is the *act of speaking*. There are special words to indicate reference to the act of speaking: *I, you, here, now, this*, as well as tenses of verbs, which determine time by reference to the time when the words are uttered. E.g. *I will go to Australia*: there is a time later than that of the utterance such that the person uttering these words goes to Australia at that time.\(^6\)

Distinction between *tokens* and *symbols*: tokens are individual signs; symbols are classes of similar symbols.

Linguistic signs must be *reproducible*. E.g. “Los Angeles is a city” and “Los Angeles is in California”. The two sentences jointly contain two tokens of the word “Los Angeles”.

Different tokens of the same symbol *have the same meaning*, or are *equisignificant*. (i) Saying so imposes the requirement that only symbols having a meaning may have tokens: so e.g. the letter “b” does not have two tokens in the expression “abba” under such a construal (it might be said to have two occurrences there instead). So here, words and sentences are said to be symbols. Also, (ii) saying so imposes a rather strong constraint on what is a word or a sentence: e.g., “I” will not be a word in the relevant sense: when uttered by distinct persons it refers to distinct persons and hence is not equisignificant on the two occasions.

In part equisignificance is given by geometrical similarity of the tokens; however also between printed, handwritten and spoken tokens there appears equisignificance. Coordination between these is a matter of convention. These tokens will be called ‘similar’. E.g. all tokens of the word “cat” belong to this very symbol (class of tokens), namely the word “cat”.

Saying “The same symbol occurs in different places (at different times)” means “Tokens of the same symbol-class appear in different places (at different times).”

*Token-reflexive* words, then, are words which refer to the corresponding token in an individual act of speech or writing.

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\(^6\) The analysis of token-reflexivity presented in the present section is based on Reichenbach’s *Elements of Symbolic Logic*, 1947, pp. 4-5, 284-7.
Uttering “dog” makes no reference to the act of utterance. Uttering “this” does. Such words are called token-reflexive.

All token-reflexive words can be defined in terms of the phrase “this token”.

- “I”: “The person who utters this token.”
- “now”: “The time at which this token is uttered.”
- “this table”: “the table pointed to by a gesture accompanying this token.”
- “John will go”: “John goes at a time later than the time at which this token is uttered.”

Let us, then, consider the (pseudo-)phrase “this token”. Let us follow H. Reichenbach (Elements of Symbolic Logic) in introducing token quotes to explain the role of “this token”.

- Ordinary quotes: applied to words, produce names of words.
- Token quotes: applied to tokens, produce tokens denoting tokens.

E.g., \( \uparrow t \uparrow \) denotes the very token of ‘t’ written between the arrow quotes, as the first occurrence of the letter ‘t’ on the above line. So

\[ \uparrow t \uparrow \]

is a token denoting the token ‘t’ in (1). [Here (1) is a name of a token, not of a symbol.]

\( \uparrow t \uparrow \) is a reflexive token, and cannot be repeated. Note, in particular, that

\[ \uparrow t \uparrow \neq \uparrow t \uparrow ! \]

Above we have two distinct tokens employing the arrow quotes, which furthermore denote distinct tokens.

What does \( \uparrow t \uparrow \) denote? A token of ‘t’. Which one? The one appearing above after the words “What does” and the initial arrow quote.

What about introducing names for tokens, like (1)? Generally, think of introducing new symbols standing for other symbols. E.g., putting \( \alpha =_{def} p \lor q \) means that
‘α’ has the same meaning as ‘p ∨ q’, i.e. that every token similar to ‘α’ is equisignificant to every token similar to ‘p ∨ q’.

Here ‘α’ stands for a class — a symbol — whose elements are tokens similar to ‘α’.

Generally, introducing a synonymous symbol involves using the relation *equisignificant* among tokens, as well as using the token-quotes operation.

Now: how to introduce a symbol synonymous to ‘t’? A *token* equisignificant with ‘t’ must be *non-reflexive*.

Let us stipulate that every token similar to ‘n’ is equisignificant to the token ‘t’. Hence the symbol ‘n’ has the same meaning as ‘t’ in single token quotes above, that is, ‘n’ is a name of the token ‘t’ above.

It should be noted that different tokens of “this token”, while similar to each other, are *not* mutually equisignificant. Therefore “this token” is rather a pseudo-phrase than a phrase.

Tenses of verbs [or rather, tensed verb forms] are a particularly important class of token-reflexive symbols: they determine time with reference to the time point of the act of speech, i.e., time point of the token uttered.

Time indication given by tenses is of a rather complex structure, as Reichenbach’s analysis in *Elements of Symbolic Logic* reveals.

## 5 Prior’s tense-logic: informal preliminaries

Let us consider A-sentences and B-sentences (cf. Sect. 3 above). In particular, B-sentences contain explicit reference to time (on a public time scale), and they are of the form

\[(1) \ p \ at \ t.\]

(An example: *Socrates is sitting at noon, December 1, 400 B.C.*) Prior did not take eternal sentences as a starting point of a logical analysis. He adopted a view according to which certain generic sentences — A-sentences — are capable of expressing complete sentential meanings, even if *not* anchored to a time scale so as to make the same statement (and have an invariant truth-value) at all times. He took token-reflexive sentences

\[(2) \ p \ (now),\]
explicitly or implicitly referring to their time of utterance, as the starting point. (An example: *Socrates is sitting.*)

It should be noted that one's understanding of sentences expressing complete propositions has general repercussions on one's logical analysis. Consider, for instance, the inference

If Dion is alive, then Dion is breathing; but Dion is alive; therefore Dion is breathing.

If the form of this inference is taken to be

“If $p$, then $q$; but $p$; therefore $q$,”

then the relevant sentences are *not* construed out of eternal sentences, but are rather understood as A-sentences. Phrased in terms of eternal sentences (having explicit time argument in each sentence), the inference would be of the form

“For any $t$, if Dion is alive at $t$, then Dion is breathing at $t$; but Dion is alive now; therefore Dion is breathing now.”

The latter reasoning would be properly studied in terms of first-order logic. Natural language apparently does not operate on the latter model. In any event, it is possible to reason in terms of A-sentences, and this is ultimately what Prior wanted to show with his tense-logic.

Let us consider examples:

(1) If Dion *will always* breath, then Dion *will* breath.

(2) If Dion *will* breath, then Dion *was to* breath.

(3) If Dion *was* alive, then Dion *will have been* alive.

It is clear how these are to be represented from the viewpoint of B-sentences (or, using first-order logic):

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7 Of course one could still argue that the *logical form* of the relevant sentences however requires the inference to be expressed in such terms: surface form of natural language is not always the most reliable guide in the logical analysis, cf., e.g., definite descriptions.

8 *Future in past*, as in ‘was [going] to sing’.

9 *Future perfect*, as in ‘will have sung’.
(1’) For any $t$, if Dion is breathing at all $s > t$, then Dion is breathing at some $s > t$.

(2’) For any $t$, if Dion is breathing at some $s > t$, then for some $s' < t$ there is $s'' > s'$ such that Dion is breathing at $s''$.

(3’) For any $t$, if Dion is alive at some $s < t$, then for some $s' > t$ there is $s'' < s'$ such that Dion is alive at $s''$.

So far so good. But how to represent these statements without resorting to the idea of sentences as eternal — in need of an explicit time indication? The solution turns essentially on how to represent contribution of tenses in A-sentences. Notably, one cannot just stay on the level of propositional logic. Think of (1), for instance. If represented in the form “If $p$, then $q$,” the proposition $p$ must be “Dion will always breath” and $q$ must be “Dion will breath.” But on the level of propositional logic such propositions $p, q$ are simply two atomic propositions, which as such are logically independent of each other (i.e., neither implies the other). Hence such an analysis of the implication (1) in terms of propositional logic does nothing by way of explicating why the implication indeed incorporates (what under certain conditions is) a valid inference. Something about the inner structure of the propositions must be brought to the fore in the logical analysis — in the case at hand something about the tense structure of the propositions (temporal information conveyed by the propositions).

Prior (Past, Present and Future, p. 17) indicates — by reference to B. Mates’s book Stoic logic (1953) and its 1955 review by P. Geach (The Philosophical Review 64, pp. 143-5) — how he came to think of presenting tenses by means of tense operators:

“Mates, in attempting to formalize the thought of Diodorus, made free use of expressions like ‘$p$ at time $t$’. (Geach [1955], reviewing Stoic Logic... naturally did not miss this, and amplified his remarks on Weinberg); I wondered if it could be done some other way, and tried writing $Fp$ for ‘It will be that $p$’, by analogy with the usual modal $Mp$ for ‘It could be that $p$’.”

Priorian tense operators will, then, syntactically be one-place functions that produce sentences out of sentence. Consider two of them:

- $F$ for ‘future’, to be read “it will be that”
• $P$ for ‘past’, to be read “it was the case that”

The operators $F$ and $P$ are called each others’ *converses* (or: *inverses*), since due to their meaning, they speak of the same relation (*earlier than*), but in converse directions: relative to time point $t$, the operator $F$ calls attention to time points $s > t$, while $P$ calls attention to time points $s < t$.

Now if ‘$p$’ stands for an A-sentence, ‘$Fp$’ likewise stands for an A-sentence. Specifically, the meaning of $Fp$ can be explained in terms of the meaning of $p$ and that of the operator $F$. What the token-reflexive sentence $Fp$ states at a time $t$ is that there be a later time $s$ such that at $s$ the sentence $p$ is true. The operator $F$ serves to introduce a ‘temporal context’ relative to the given temporal context $t$ (namely one which is otherwise unspecified but must satisfy $s > t$). Sentence $Fp$ is true at $t$ if there is at least one $s$ later than $t$ such that $p$ holds at $s$. Note that the metalanguage truth condition of the A-sentence $Fp$ is in effect a B-sentence.

At least on the surface level the Priorean analysis of tensed sentences is closer to natural language than an analysis in terms of B-sentences. Namely, natural language tenses do not introduce anything similar to bound variables of first-order logic (cf. the $t, s, s', s''$ in ($1'$), ($2'$) and ($3'$) above).

Before moving on, and to facilitate further discussion, it is useful to introduce *duals* of the operators $F$ and $P$:

• $G$ for ‘it is always going to be’, to be read “always in the future, it is the case that”

• $H$ for ‘it has always been’, to be read “always in the past, it is the case that”

$P$ and $H$ are related like existential and universal quantifier of first-order logic. The same holds for the relationship of $F$ and $G$: just like universal quantifier can be defined in terms of existential quantifier and negation ($\forall x = \neg\exists x\neg$), $H$ can be defined as $\neg P\neg$, and $G$ can be defined as $\neg F\neg$. (Of course the definability would work in the other direction, too: e.g. $P$ could be defined as $\neg H\neg$.) Like $F$ and $P$, also $G$ and $H$ can be called each others’ *converses*. 